M. Math II yr / Second Semester / Lie groups.
2008 – 2009

1) Let (π, V) be a representation of G. What is meant by saying $v \in V$ is a cyclic vector for this representation? If every $v \neq 0$ is a cyclic vector, prove that π is irreducible.

2) Identify $SU(2)$ with S^3. Define the function f on S^3 by:

$$f(x_1, x_2, x_3, x_4) = x_4^2 + x_3^2 + x_2^2 - 2x_1^2$$

Evaluate $\int_{SU(2)} f \, d\mu$ where μ is the normalized Haar measure.

3) Let G be a finite group. Prove that the number N of inequivalent irreducible (unitary) representations of G is finite. If G is abelian what is N?

4) What is the Haar measure μ of $GL^+(2, \mathbb{R})$? (Writing a general element $x = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$, write down the Haar measure explicitly.)

5) Let π be a representation of (a compact) G on V. If the multiplicity of the trivial one dimensional representation of G in π is <u>zero</u>, what is the value of $\int_G f \, d\mu$ for $f \in \mathcal{R}_\pi$? (Here μ is the Haar measure on G.)

You may use theorems proved in class or in the book by B-M-S-T, However quote the theorems precisely and show how they are used.